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Measurement of Dielectric Parameters at Microwave Frequencies by Cavity-Perturbation Technique

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***Abstract*—Relations for evaluating dielectric parameters from changes in resonance frequency and Q of a cylindrical TM_{010}-mode cavity have been derived for thin samples of length less than the height of the cavity. Although derived under some simplifying assumptions, they reduce to the standard form when the length of the specimen equals the height of the cavity, and yield consistent results when applied to different lengths of the same material.**

I. Introduction

THE CAVITY perturbation technique has been extensively employed for studying the dielectric and magnetic properties of materials at microwave frequency [1]–[4]. The method has its own advantages but it has certain limitations, especially the requirement that the volume of the specimen must be small so as to produce a negligible effect on the field configuration in the cavity. Also the sample must be in one of the specified shapes for which formulas have been worked out. For the case of a cylindrical cavity resonating in the TM_{010} mode, the sample used is mostly in the form of a thin rod, the length of which equals the height of the cavity, so that both the ends of the specimen are in contact with the cavity walls [5]. The quality factor Q of the cavity depends upon its height and it is convenient to work with high-Q cavities. However, often it becomes difficult to machine long and thin samples. A need, therefore, arises for extending the cavity-perturbation technique to include specimens of length less than the height of the cavity. In this article a set of formulas, based on certain assumptions, have been developed by calculating an effective depolarizing factor in order to meet this requirement. The resulting equations for dielectric parameters have been found to be adequate in yielding consistent results when applied to specimens of different lengths of the same material. Moreover, these formulas readily reduce to the ones used for full-size specimen under the condition that the length of the specimen is equal to the height of the cavity.

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II. Theory

A. Relations for a Prolate Ellipsoid of Insufficient Length Held Coaxially in a Cylindrical Cavity Resonating in the TM_{010} Mode

Waldron [5], [6] has shown that relative change in the complex resonance frequency due to the insertion of a small specimen in a resonating cavity bounded by perfectly conducting surface is given by

$$\left[\frac{\delta\Omega}{\omega_0}\right] = \frac{\iiint_{V_0}\{(\vec{E}_1\cdot\vec{D}_0 - \vec{E}_0\cdot\vec{D}_1) - (\vec{H}_1\cdot\vec{B}_0 - \vec{H}_0\cdot\vec{B}_1)\}\,dV}{\iiint_{V_0}\{\vec{E}_0\cdot\vec{D}_0 - \vec{H}_0\cdot\vec{B}_0\}\,dV} \tag{1}$$

where $[\delta\Omega/\omega_0]$ is related to the changes in the resonance frequency and the quality factor Q of the cavity, through the relation

$$\left[\frac{\delta\Omega}{\omega_0}\right]=\frac{\delta\omega}{\omega_0}+\frac{j}{2}\left(\frac{1}{Q_1}-\frac{1}{Q_0}\right) \tag{2}$$

Q_0 and Q_1 being the quality factors of the empty and cavity with sample, respectively; V_0 is the volume of the cavity; and $\vec{E}_0$, $\vec{H}_0$, $\vec{B}_0$, and $\vec{D}_0$ are the fields in the unperturbed cavity. The quantities with subscript "1" are the perturbations caused by the specimen which are assumed to be important only in the volume occupied by the specimen, and have been totally neglected outside the specimen in comparison with the corresponding quantities in the unperturbed cavity while deriving (1).

Expressions for dielectric parameters are obtained by evaluating the right-hand side of (1) using certain approximations [2], [6]. In the approximation used by Waldron, the fields in the perturbed case are equated to the sums of the unperturbed fields ($\vec{E}_0, \vec{H}_0$) and additional fields ($\vec{E}_1, \vec{H}_1$) exclusively, due to the specimen. Thus the fields in the perturbed cavity are

$$\vec{E}_2=\vec{E}_0+\vec{E}_1$$
$$\vec{H}_2=\vec{H}_0+\vec{H}_1. \tag{3}$$

In cylindrical coordinates (γ,θ,z), the components of $\vec{E}_0$ and $\vec{H}_0$ for the TM_{010} mode can be expressed as [5]

$$E_\gamma=E_\theta=0$$
$$E_z=J_0(k\gamma)$$
$$H_\gamma=H_z=0$$
$$H_\theta=j\frac{\omega\varepsilon_0}{k}J_1(k\gamma) \tag{4a}$$

with

$$k^2=\omega^2\varepsilon_0\mu_0. \tag{4b}$$

$J_0(k\gamma)$ and $J_1(k\gamma)$ are the zeroth- and first-order Bessel functions, respectively; k is the wavenumber; and ε_0 and μ_0 are the permittivity and permeability of the free space.

The added or partial fields $\vec{E}_1$ and $\vec{H}_1$ inside the specimen are essentially the depolarizing and the demagnetizing fields, which in terms of the polarization $\vec{P}_0$ and magnetization $\vec{M}_0$, respectively, are given by [7]

$$\vec{E}_1=-N\vec{P}_0 \tag{5a}$$
$$\vec{H}_1=-N\vec{M}_0 \tag{5b}$$

where N being the depolarizing (or demagnetizing) factor, depends on the axial ratio of the specimen. Values for N have been calculated by many workers [8]–[10]. Following Bozorth, N for a prolate ellipsoid with semiaxes h, b, and c $(b=c)$, polarized along the longer axis, is given by

$$N=\frac{1}{(m^2-1)}\left[\frac{m}{2(m^2-1)^{1/2}}\ln\frac{m+(m^2-1)^{1/2}}{m-(m^2-1)^{1/2}}-1\right] \tag{6}$$

where m is the ratio h/b. The factor N so far has been calculated for dielectric specimen in the absence of any conducting surface near the specimen. For dielectric specimen held in a resonant cavity, the depolarizing factor N and hence, the depolarizing field $\vec{E}_1$ are likely to get modified especially when the length of the specimen is less than the height of the cavity. The modified depolarizing field $\vec{E}_1$ may be calculated in the following way.

B. Evaluation of $\vec{E}_1$

To evaluate $\vec{E}_1$, the following additional assumptions have been made besides the usual that the perturbation caused by the specimen is too small to disturb the field configuration in the cavity.

1) The standing microwave is replacable by a static field.

2) The specimen acts like a dipole of length equal to the length of the specimen and, as suggested by the method of images, forms an infinite number of dipoles A_iB_i $(i=1,2,\cdots,\infty)$, as shown in Fig. 1.

3) For each image dipole like A_1B_1, formed by the image charges, the magnitude of the total polarization is equal to that of the specimen P_0V_1, given by

$$P_0V_1=(\epsilon_1^*-1)\epsilon_0E_2V_1 \tag{7}$$

with

$$\epsilon_1^*=\epsilon_1'-j\epsilon_1''$$

where V_1 is the volume of the specimen.

4) The effect of the charges induced in the cylindrical surface of the cavity is negligible.

Waldron [11] has used the approximation 1) in the situation where the sample thickness is much smaller than the wavelength of the microwave signal, and the region where the sample is situated is in uniform field. These conditions are satisfied in the present case also. The second assumption simplifies the evaluation of the effect of charges induced in the flat surfaces of the cavity as it ignores the surface distribution of charge of the specimen. In other words, a prolate ellipsoid can be replaced by a cylinder of equal volume. This assumption is not likely to cause much error for thin samples. The basis for the third assumption is that the calculation of total polarization (dipole moment) of the dipole, like A_1B_1 (Fig. 1), formed by the image charges as product of charge and length leads to ever-increasing values if each of the induced charges be considered equal in magnitude to the one appearing at the top (or bottom) of the specimen. The polarization of the specimen attaining infinite value due to the presence of cavity's walls is, however, physically unacceptable. Assumption 3) on the other hand does not lead to such unphysical results. The last assumption is based on the cylindrical symmetry prevailing in the cavity.

If the length and area of the cross section of the sample be $2h$ and A, respectively, and the height of the cavity be $2H$, the magnitude of the polarization (dipole moment per

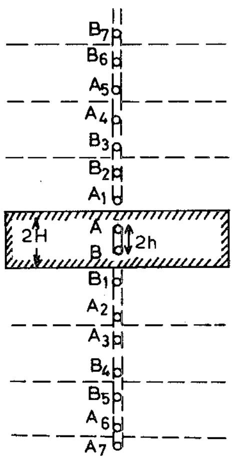

Fig. 1. Formation of image dipoles (like A_1B_1) of the specimen AB, held coaxially in a cylindrical cavity.

unit volume) of the first image dipole A_1B_1 will be

$$P_1=\frac{P_0V_1}{(4H-2h)A}=P_0\frac{h}{2H-h}.$$

The net depolarizing field due to the polarization of the sample and its image dipoles can be written as

$$\begin{aligned}\vec{E}_1&=-N\sum_{n=0}^{\infty}(-1)^n\vec{P}_n\\&=-N\vec{P}_0\left[\left(1-\frac{h}{2H-h}\right)+\left(\frac{h}{2H+h}-\frac{h}{4H-h}\right)+\cdots+\infty\right]\\&=-N\vec{P}_0h\sum_{n=0}^{\infty}\left[\frac{1}{(2nH+h)}-\frac{1}{(2n+2)H-h}\right]\\&=-N\vec{P}_0\left[1-2\sum_{n=1}^{\infty}\frac{1}{(nc)^2-1}\right]\end{aligned}$$

where c has been used for the ratio $2H/h$. The use of the standard result [12]

$$\sum_{n=1}^{\infty}\frac{1}{(z^2-n^2)}=\frac{\pi}{2z}\cot(\pi z)-\frac{1}{2z^2}$$

leads to

$$\vec{E}_1=-\left[N\frac{\pi h}{2H}\cot\frac{\pi h}{2H}\right]\vec{P}_0. \tag{8}$$

On comparing (5a) and (8) it is seen that the effect of the images is to alter the depolarizing factor N to

$$N_e=N\frac{\pi h}{2H}\cot\frac{\pi h}{2H} \tag{9}$$

N_e may be termed as the effective depolarizing factor.

To investigate the nature of N_e, a set of graphs of N_e versus h/H has been plotted for various values of m in Fig. 2. It is seen that for a given m, the effective depolarizing factor decreases with increasing h/H and approaches zero when h tends to H. Moreover in the region not included in Fig. 2, as h tends to zero, N_e tends to unity as it should, because when m is zero the depolarizing factor as given by (6) is unity.

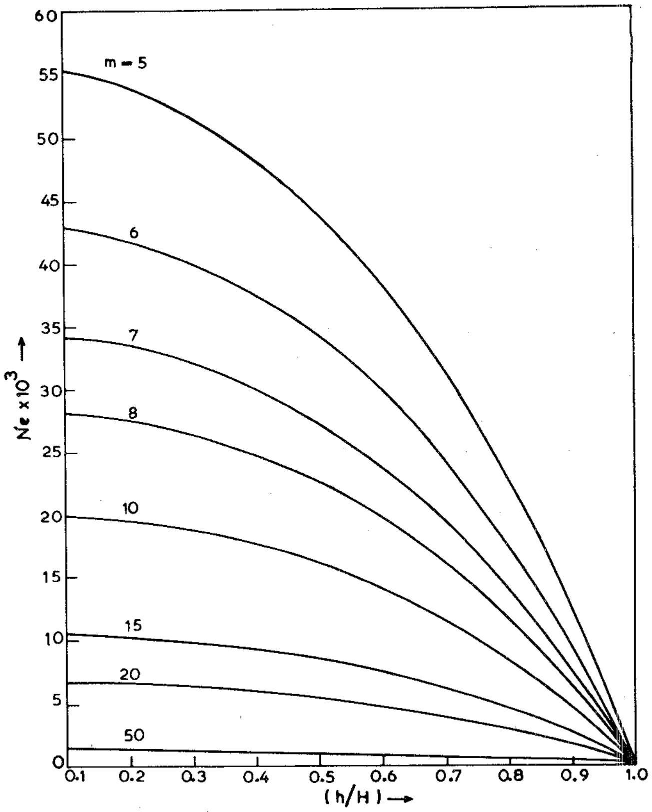

Fig. 2. Variation of the effective depolarizing factor (N_e) with the ratio of the length of the specimen to the height of the cavity (h/H).

III. Relations for Dielectric Parameters

Substituting for $\vec{E}_2$, $\vec{E}_0$, $\vec{H}_0$, and $\vec{E}_1$ from (3)–(8) taken along with (9) in (1), integration and separation of the resulting equation into real and imaginary parts gives

$$\epsilon_1'-1=\frac{V_0\frac{\delta\omega}{\omega_0}\left[\frac{V_1}{2J_1^2(ka)}-N_eV_0\frac{\delta\omega}{\omega_0}\right]-N_eV_0^2\left[\delta\left(\frac{1}{2Q}\right)\right]^2}{\left[\frac{V_1}{2J_1^2(ka)}-N_eV_0\frac{\delta\omega}{\omega_0}\right]^2+\left[N_eV_0\delta\left(\frac{1}{2Q}\right)\right]^2} \tag{10}$$

$$\epsilon_1''=\frac{V_0\delta\left(\frac{1}{2Q}\right)\left[\frac{V_1}{2J_1^2(ka)}-N_eV_0\frac{\delta\omega}{\omega_0}\right]+N_eV_0^2\frac{\delta\omega}{\omega_0}\delta\left(\frac{1}{2Q}\right)}{\left[\frac{V_1}{2J_1^2(ka)}-N_eV_0\frac{\delta\omega}{\omega_0}\right]^2+\left[N_eV_0\delta\left(\frac{1}{2Q}\right)\right]^2} \tag{11}$$

for dielectrics with $\mu_1=1$ and placed in a cylindrical cavity of radius a, where H_0 is zero. $\delta(1/2Q)$ has been used for the difference $\frac{1}{2}((1/Q_1)-(1/Q_0))$.

The electric field at the walls of the cavity must be zero. Therefore, it is implied from (4a) that

$$J_0(ka)=0$$

or

$$ka=2.40483, 5.2008,\cdots,\cdots.$$

For a first-order mode of excitement of the cylindrical cavity, $1/2J_1^2(ka)$ is about 1.8552. The measurements of the quantities $\delta\omega/\omega_0$ and $\delta(1/2Q)$ and lengths and volumes of the specimen and cavity can thus provide the values of ϵ_1' and ϵ_1'' from (10) and (11), respectively.

Equations (10) and (11) reduce to the following standard form:

$$\epsilon_1' = 1 + 0.539 \frac{V_0}{V_1} \frac{\delta\omega}{\omega_0}$$

$$\epsilon_1'' = 0.539 \frac{V_0}{V_1} \delta\left(\frac{1}{2Q}\right) \tag{12}$$

for the case when the length of the sample is the same as the height of the cavity. It may be noted that for $h = H$, the depolarizing field becomes zero as expected [13].

IV. Results and Discussions

The relations (10) and (11) have been obtained under some simplifying assumptions. To test the validity of the approximations and the derived relations, measurements have been made on some samples of different lengths at 3.698 GHz. The resonant cavity used was of medium quality factor ($Q_0 \simeq 1800$), a radius of 3.1 cm, and height 2.0 cm. The measurement technique was essentially the same as has been described earlier [4].

Each sample was held firmly and symmetrically along the axis of the cavity in a hole drilled in a very thin and narrow strip of mica. The mica strip was placed in such a way that it passed through the centre of the cavity. The sample was inserted by removing the lid of the cavity. The cavity used was designed with a step in its cylindrical wall which allows a larger area of contact with the lid, and it was ensured that the lid occupies the same position relative to the cavity every time, so that the cavity characteristics may repeat within the limits of experimental errors.

The mica strip was found to have little effect on the cavity resonant frequency and quality factor as compared to the empty cavity values. However, for calculating the dielectric parameters, the resonant frequency and quality factor of the empty cavity have been taken to be those with the mica strip and without any sample.

The samples used were Teflon (area of cross section: 0.0950 cm^2), ferrite ($Li_{0.5}Mn_{0.1}$, $Fe_{2.4}O_4$; area of cross section: 0.0053 cm^2) and $LaCrO_3$ (area of cross section: 0.0116 cm^2). The last two were in ceramic form, and it was ensured that different lengths of the samples had the same density.

The effective depolarizing factor N_e has been calculated by considering a prolate ellipsoid equal in volume to that of the sample and with its major axis equal to the length of the sample.

The results are tabulated in Table I. The values calculated from (12) have also been included to show the magnitude and extent of the correction term. It was noted that the use of N for N_e gives inconsistent results for different lengths of the same material. The room temperature dielectric constant of $LaCrO_3$ is in good agreement with the reported low-frequency value 10 at 77 K [14]. The dielectric constants of Teflon and ferrite measured for different lengths show very good agreement with the one obtained for full-size sample. The "loss" of later specimens could not be measured because of the limitations in the accurate measurement of changes in frequencies. It may be pointed out that good agreement is obtained for different lengths of the sample, although in case of ferrite the smallest length is only 24 percent of the height of the cavity. However, the smallest length of a sample may be limited by the approximations used in deriving the relations (10) and (11). Especially for very small lengths the curvature of the fringing field may disturb the uniformity of the field outside the sample. However, the present experimental results show that for a sample of dielectric constant 14, the relations are valid even though the length of the sample is as small as about one-fourth of the height of the cavity. The good agreement between the measured values for different lengths of a sample indicates that the approximations involved in deriving the relations (10) and (11) are justified.

TABLE I
Dielectric Constants and Loss Calculated From (12), (10), and (11) at Room Temperature (35°C) and Frequency 3.6986 GHz (Q_0=1849)

Material	Length of the sample (2h) cms	Change in Res. frequency MHz	Loaded quality factor Q_L	from Eqs (12) ϵ_1'	from Eqs (12) ϵ_1''	from Eqs (9) and (10) ϵ_1'	from Eqs (9) and (10) ϵ_1''
Teflon	2.00	23.0	1837	2.07	–	2.07	–
Teflon	1.50	16.5	1841	2.03	–	2.05	–
Teflon	1.05	11.5	1843	2.01	–	2.08	–
Teflon	0.60	5.5	1846	1.89	–	2.03	–
Ferrite+	2.00	16.5	1841	14.10	–	14.10	–
Ferrite+	1.45	9.0	1844	13.41	–	13.90	–
Ferrite+	0.95	6.0	1846	12.27	–	13.70	–
Ferrite+	0.48	2.5	1848	10.29	–	14.40	–
LaCrO++	0.80	8.0	384	8.22	3.44	9.40	5.28
LaCrO++	0.56	4.5	672	7.36	2.48	9.50	4.86

+ $Li_{0.5}Mn_{0.1}Fe_{2.4}O_4$
++ Lanthanum chromite

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The Effect of Flange Loss on the Reflection Coefficient of Reduced-Height Waveguide Reflection Standards

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***Abstract*—It is shown that when reflection-coefficient standards are constructed from reduced-height waveguide having the plane of the change in height coincident with that of the flange, the flange loss will reduce the calculated reflection coefficient, and for small reflection-coefficient standards ($|\rho_S| < 0.1$) and practical flange losses, the effect is more significant than step capacitance which is usually taken into account.**

I. Introduction

FOR AN international intercomparison of waveguide reflection-coefficient magnitude at 10 GHz between a number of standards laboratories[1], National Bureau of Standards (NBS), United States, and Royal Signals and Radar Establishment (RSRE), United Kingdom, have submitted travelling standards to be measured by all participants. During the course of this intercomparison the extent to which these reflection-coefficient standards are "absolute", i.e., calculable from dimensions, was examined. It has been found that the ever-present flange loss significantly affects the calculated performance of the reduced-height travelling standards.

II. Discussion

The most frequently used fixed waveguide reflection-coefficient standard takes the form of a nominal width, but reduced height waveguide, terminated in a sliding matched load. Sliding the (imperfect) matched load results in a circular movement of the input reflection coefficient on the Smith chart, and the center of this circle represents the reflection that would be obtained with a perfect termination. The factors usually taken into consideration in predicting the reflection from such a device are as follows.

1) The change in characteristic impedance resulting from the reduced height of the guide, where $z_0 \propto b'/b$, and b' and b are the reduced and nominal heights of the guides on the two sides of the flange connection.

2) The step capacitance resulting from the setting up of evanescent modes by the abrupt change of guide height.

3) The radii of curvature of the corners of the nominally rectangular waveguide aperture.

However, flange loss also will affect the reflection coefficient, but hitherto this effect usually has been neglected.

As evidenced below, flange loss may be modeled as a series, lumped resistance, and its normalized value is given by

$$R = 2(10^{A/20} - 1) \tag{1}$$

where A is the attenuation in decibels caused by the series insertion of R between a source and load, both having normalized resistances of unity. The reflection coefficient of a normalized series resistance of R is

$$\rho_R = \frac{R}{2+R} \tag{2}$$

and it may be shown that if a standard having a reflection

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